

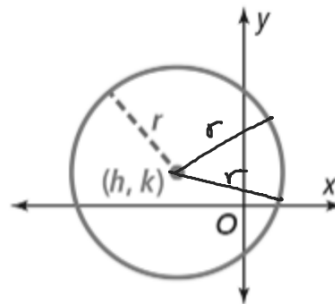
Equation of a Circle

An equation of a circle with center (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2.$$

PROOF: SEE EXERCISE 13.

If...



Then... $(x - h)^2 + (y - k)^2 = r^2$

radius
radi

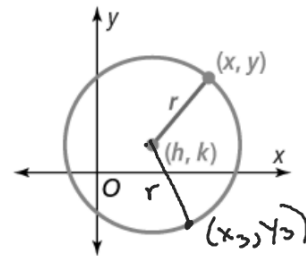
Use variables that can apply to any circle on the coordinate plane. Draw a circle with point (h, k) as the center of the circle. Then select any point (x, y) on the circle.

Use the Distance Formula to find the distance r between the two points.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$(x - h)^2 + (y - k)^2 = r^2$$

Because the radius is the same from the center to any point (x, y) on the circle, this equation satisfies all points of the circle.



1. What are the radius and center of the circle with the equation

$$(x - 2)^2 + (y - 3)^2 = 25?$$

$$\text{Center } (2, 3)$$

$$(x-h)^2 + (y-k)^2 = r^2$$

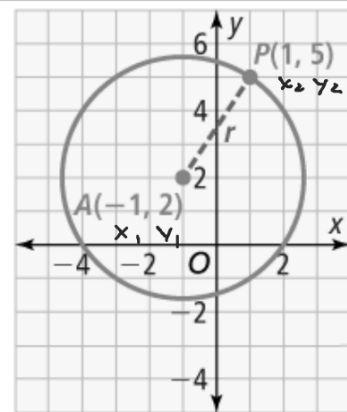
$$r = \sqrt{25} = 5$$

What is the equation for $\odot A$?

$$(x-h)^2 + (y-k)^2 = r^2$$

SOLUTION

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1 - (-1))^2 + (5 - 2)^2} \\ &= \sqrt{(2)^2 + (3)^2} \\ &= \sqrt{4 + 9} = \sqrt{13} \end{aligned}$$

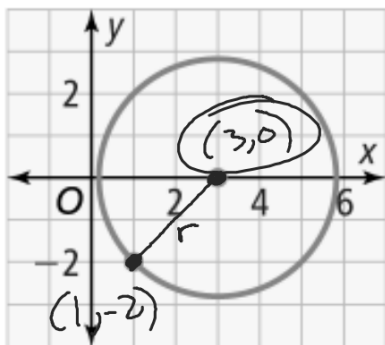


$$C(-1, 2) \quad r = \sqrt{13}$$

$h \quad k$

$$(x+1)^2 + (y-2)^2 = 13$$

2. a. What is the equation for the circle? $(x-h)^2 + (y-k)^2 = r^2$



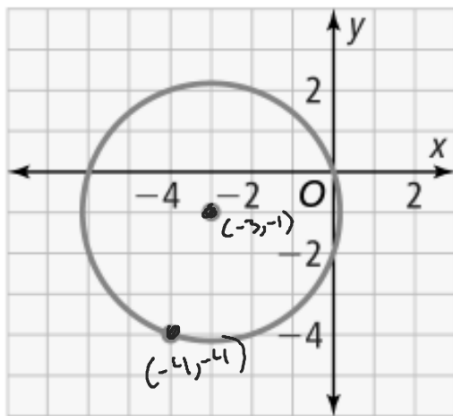
$$\begin{aligned} r &= \sqrt{(3-1)^2 + (0-(-2))^2} \\ &= \sqrt{2^2 + 2^2} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \end{aligned}$$

$$\begin{aligned} &\sqrt{(1-3)^2 + (-2-0)^2} \\ &= \sqrt{(-2)^2 + (-2)^2} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \end{aligned}$$

$$(x-3)^2 + y^2 = 8$$

$$\begin{aligned} r &= \sqrt{8} \\ r^2 &= (\sqrt{8})^2 \\ &= 8 \end{aligned}$$

2. b. What is the equation for the circle?



$$(h, k) = (-3, -1)$$

$$r = \sqrt{(-3 - (-4))^2 + (-1 - (-4))^2}$$

$$= \sqrt{1^2 + 3^2}$$

$$= \sqrt{10}$$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x + 3)^2 + (y + 1)^2 = 10$$

Circle Q has radius 7 and is centered at the origin. Does the point $(-3\sqrt{2}, 5)$ lie on $\odot Q$?

x y

$$x^2 + y^2 = 7^2$$

$$x^2 + y^2 = 49$$

$$(-3\sqrt{2})^2 + (5)^2 \stackrel{?}{=} 49$$

$$18 + 25$$

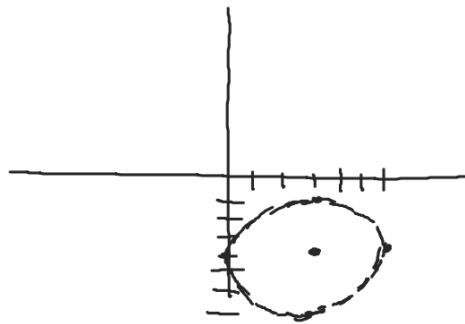
$$43 \neq 49$$

Not on Circle

What is the graph of $(x - 3)^2 + (y + 4)^2 = 9$?

Center $(3, -4)$ $r = 3$

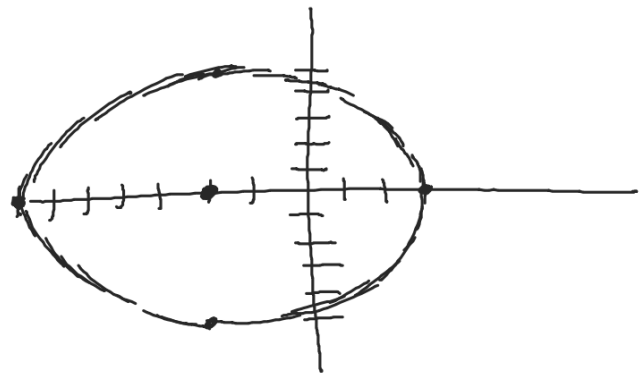
$$r^2 = 9$$



4. What is the graph of the circle?

Center $(-2, 0)$ $r = 5$

a. $(x + 2)^2 + y^2 = 25$



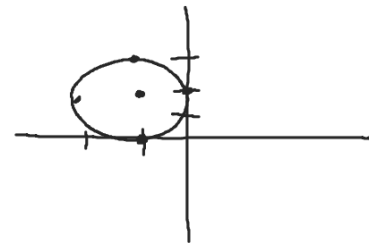
4. What is the graph of the circle?

b. $(x + 1)^2 + (y - 2)^2 = 1$

.....

Center $(-1, 2)$

$r = 1$



5. What are the center and radius of the circle with equation

$$(x - 4)^2 + (y - 9)^2 = 1?$$

$$C (4, 9)$$

$$r = 1$$

6. What is the equation for the circle with center $(\overset{h}{6}, \overset{k}{2})$ and radius $\overset{r}{8}$?

$$(x-6)^2 + (y-2)^2 = 64$$

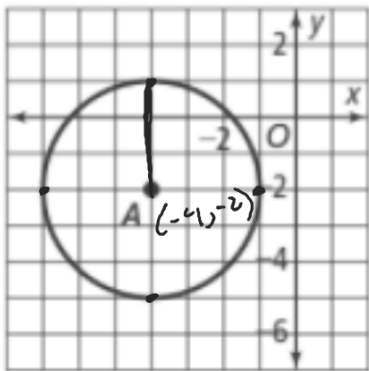
8. What is the equation for the circle with center $(-9, 5)$ and radius 4?

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x+9)^2 + (y-5)^2 = 16$$

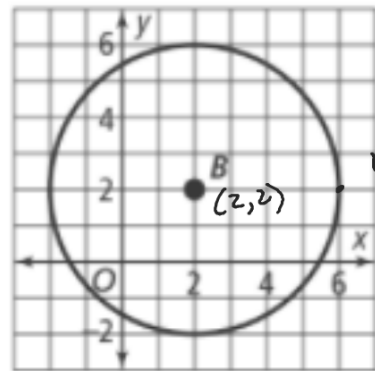
Write the equation for the circle shown in each graph.

3.



$$(x+4)^2 + (y+2)^2 = 9$$

4.



$$(x-2)^2 + (y-2)^2 = 16$$

Write the standard form of the equation of the circle with the given characteristics. endpoints of a diameter: (- 1, 4) , (7, 6)

$$(x-h)^2 + (y-k)^2 = r^2$$